REYNOLDS RELAXATION OF FRICTIONAL STRESS IN NONEQUILIBRIUM TURBULENT BOUNDARY LAYER

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Reynolds relaxation of frictional stress is analyzed in a nonequilibrium turbulent boundary layer with sign reversing longitudinal pressure gradient.

Relaxation Phenomena in Turbulent Boundary Layer. In recent studies of turbulent boundary layers, there appears a distinct trend away from the use of Boussinesq's classical relation for determining the local Reynolds frictional stress $\tau = -\rho \langle u'v' \rangle$ and toward the use of new relaxational relations between frictional stress and average-velocity field instead. An example is the Hinze relaxation equation [1]

$$L_x \frac{\partial \tau}{\partial x} + \tau = \mu_t \frac{\partial u}{\partial y}, \qquad (1)$$

free of local boundedness. Without the $L_x \partial \tau / \partial x$ on the left-hand side, this equation becomes the Boussinesq equation for local stress. According to the relaxation equation, the magnitude of the Reynolds frictional stress at some point depends on magnitudes of the Reynolds stress in the upstream region preceding that point and thus on the "history" of the stream. The relaxation equation, therefore, expresses the ability of a "vortical" structure of turbulence to "memorize" the preceding flow conditions.

In Eq. (1) we have L_x denoting the path length of longitudinal relaxation, here assumed to be $L_x = ax$ (value of the proportionality factor a is, according to experimental data [2], within the 0.3-0.4 range), and μ_t denoting the eddy dynamic viscosity according to Boussinesq. The relaxation equation (1) can be derived from the equation of stress transfer for Reynolds frictional stress, with several simplifications. An essential factor is that the equation has a solution in quadratures

$$\tau = \tau_0 \exp\left(-\int_{x_0}^{x} \frac{dx'}{L_x(x')}\right) + \int_{x_0}^{x} \frac{\mu_t \partial u/\partial y}{L_x(x')} \exp\left(-\int_{x'}^{x} \frac{dx''}{L_x(x'')}\right) dx', \qquad (2)$$

where x_0 and τ_0 are, respectively, the abscissa and the Reynolds frictional stress corresponding to y in the initial section of the boundary layer [2, 3].

The path length L_x of longitudinal relaxation has already been determined experimentally and the validity of the relaxation Eq. (1) has been verified in specific cases [2]. In that study [2], for instance, the relaxation of average and fluctuation characteristics of a stream in a turbulent boundary layer at longitudinally streamlined plate was examined. A half-sphere resting on the plate surface, with a radius equal to approximately 40% of the local thickness of the boundary layer, served as barrier perturbing the stream. Measurements of the Reynolds stress at various distances from the plate surface revealed that the relaxation path had become longer upon transition from inner to outer region of the turbulent boundary layer, with the Reynolds stress perturbed by the half-sphere returning to its unperturbed level at correspondingly farther abscissas x. Thus, by measuring the turbulence characteristics of a stream in the process of relaxation, P. J. H. Builtjes confirmed the fundamental conclusion at which F. Klauser had arrived in 1959 already [4] on the basis of measurements of relaxing velocity profiles in a turbulent boundary layer at a plate with a cylinder serving as perturbator. Klauser's theory reduces to the following: the outer region of a boundary layer with large "vortices" returns to its unperturbed state slower than the inner region with "smaller" vortices does. This means the ability of turbulent "vortices" to "memorize" the preceding states of a stream increases as their scale increases. This trend is further confirmed in the more intricate case of a nonequilibrium turbulent boundary layer with sign reversing pressure gradient.

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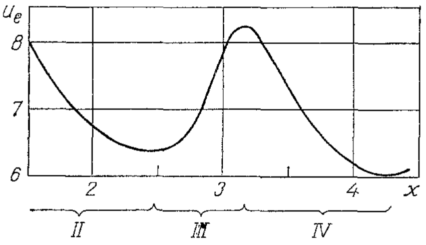

Fig. 1. "Outer" velocity u_e (m/sec) as function of longitudinal coordinate x (m): II, IV) diffuser zones; III) nozzle zone.

"Memory" Effects in Turbulent Boundary Layer with Sign Reversing Pressure Gradient. The author has evaluated the results of measurements, wide in scope and quite accurate, made by the Japanese scientists Tsuij and Morikawa [5] in experiments involving a turbulent boundary layer with pressure gradient. A longitudinal distribution of the pressure gradient was established by means of a flexible wall shaped so as to produce a boundary layer without separation in the diffuser zones and without restoration of laminar flow in the nozzle zones. The corresponding velocity u_e distribution along the outer edge of the boundary layer is shown in Fig. 1. Study [5] contains experimental data only, without theoretical discussion. For determining the Reynolds frictional stress according to the Boussinesq "local" relation, this author used the two-layer method of stipulating the eddy dynamic viscosity according to Prandtl $\mu_t = \rho \varkappa^2 y^2 \partial u/\partial y$ ($\varkappa = 0.4$) in the inner region and according to Klauser $\mu_t = \rho K u_e \delta^*$ ($K = 0.0168$) in the outer region of the boundary layer [6]. The derivative $\partial u/\partial y$ was determined through graphical differentiation of experimental velocity profiles in sections of the boundary layer. After the local Reynolds frictional stresses had been determined, "relaxation" analogs were calculated according to relation (2). As the initial value τ_0 was used as the experimental one in the first measured τ-profile (x = 1.9 m) with the corresponding value of the dimensionless distance from the wall. The path length of longitudinal relaxation was stipulated according to the relation $L_x = 150\delta^{**}$, which in the absence of a pressure gradient coincides with the Builtjes length $L_x = 0.31x$ [2]. The graph in Fig. 2 depicts longitudinal distributions of the dimensionless Reynolds frictional stress in the inner region ($y/\delta = 0.1$) and in the outer region ($y/\delta = 0.7$), calculated either according to the relaxation theory (solid lines) or according to the Boussinesq relation (dashed lines) and based on experimental data according to Tsuij–Morikawa (dots).

An analysis of the data in Fig. 2 reveals that the Boussinesq "local" relation predicts the distribution of stress τ with sufficient accuracy in the nozzle zone III but inaccurately in the diffuser zones II and IV, also that the agreement between stress τ, calculated according to the relaxation theory and determined experimentally improves upon transition from inner region (Fig. 2a) to outer region (Fig. 2b) of the boundary layer. With transition from a nozzle zone to a diffuser zone, therefore, the relative influence of "memory" effects on the shape of the profiles of the Reynolds frictional stress increases from inner region to outer region. This is attributable to the increase of the characteristic dimension of turbulent "vortices" as well as to the attendant trend toward a more uniform average-velocity field, i.e., the decrease of the derivative $\partial u/\partial y$. The more uniform the average-velocity field becomes, the more pronounced become nonlocal effects of the "memory" of turbulent "vortices." In calculations pertaining to turbulent boundary layers with relatively large "vortices" and small nonuniformity of the average-velocity field, therefore, the Boussinesq "local" relation must be replaced with more intricate relations taking into account the flow "history."

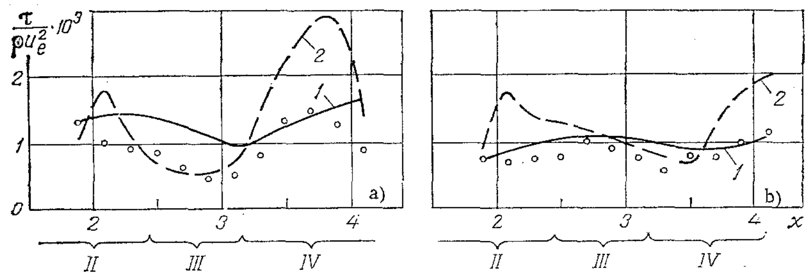

Fig. 2. Dimensionless Reynolds frictional stress as function of the x-coordinate (m): (a) $y/\delta = 0.1$; (b) $y/\delta = 0.7$: 1) according to solution to Eq. (1); 2) according to Boussinesq relation; dots represent experimental data.

NOTATION

τ, Reynolds frictional stress; < >, averaging in time; ρ, density; x, longitudinal coordinate; y, transverse coordinate; u', longitudinal fluctuation velocity; v', transverse fluctuation velocity; u, average longitudinal velocity; L_x, path length of longitudinal relaxation; μ_t, eddy dynamic viscosity; $\varkappa$, K, a, empirical constants; x', x", integration variables in relation (2); subscript 0 refers to the value of a quantity in the initial section; u_e, average velocity at the outer edge of a boundary layer; δ^*, displacement thickness; δ^{**}, momentum thickness; and δ, local thickness of a boundary layer.

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CORRELATION BETWEEN AMPLITUDES OF HARMONIC COMPONENTS OF VELOCITY PULSATIONS

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A correlation is empirically established between the amplitudes of harmonic components of velocity pulsations of a turbulent flow next to a wall.

In analyzing histograms of velocity pulsations for turbulent steady-state flow, the flow regime is often modeled as a steady, stochastic process. This makes it possible to represent it as a set of elemental harmonic components. Here, it is assumed that the harmonic components are not coupled to each other (do not correlate), so that, within the framework of this model, it is sufficient to find the dispersion of the components (spectra). The sum of these components, meanwhile, equals the power of the process.

In studying the mechanism by which energy is transferred from one perturbation to another, it is of interest to know not only the dispersion of the components, but also the parameters of their interaction. Here, we note that the possibility of a connection existing between the components is probabilistic rather than rigorous (determined), since energy is divided and transferred at random moments of time and the division takes place on structures with random dimensions.

As concerns correlations between harmonic components, it is necessary to regard the process as unsteady, i.e., its characteristics will depend on time, and the sum of the dispersions of the components (spectra) will not be equal to the power of the process, since part of this power is spent on the interaction between the components. The periods of transience which occur due to the correlation between the components may be comparable to the periods of these components, so that they cannot be detected by the time-averaging methods of analysis which are widely used.

Harmonizable processes [1-3], which can be grouped into several classes, may serve as a model of a signal representing velocity pulsations in steady flow which will allow for a correlation between the (harmonic) components. Dragan introduced the class of D-harmonizable processes, the criterion of which is a finite total dispersion of the components. This condition is satisfied for velocity histograms.

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